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Different Modelling and Controlling Technique For Stabilization Of Inverted Pendulam

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Abstract—In this paper modeling of an inverted pendulum has been done and then two different controllers (PID & SFB) have been used for stabilization of the pendulum. The proposed system extends classical inverted pendulum by incorporating two moving masses. The motion of two masses that slide along the horizontal plane is controllable .The results of computer simulation for the system with Proportional, Integral and Derivative (PID) & State Feedback Controllers are shown.

Index Terms-Inverted pendulam,swing up control,nonlinear control, gain formulae, pole placement,PID controller,State Feedback controller.

1 INTRODUCTION

An inverted pendulum is one of the most commonly studied system in the control area[6]. The control objective of the inverted pendulum is to swing up the pendulum hinged on the moving cart by a linear motor from stable position (vertically down state) to the zero state(vertically upward state) and to keep the pendulum in vertically upward state in spite of the disturbance[1].

In the field of engineering and technology the importance of benchmark[11] needs no explanation. They make it easy to check whether a particular algorithm is giving the requisite results.

A lot of work has been carried out on the inverted pendulum in terms of its stabilization. Many attempts have been made to control it using classical control ([2],[4]). Thus it is a knowledge which has been widely used in the study of stabilization of space vehicle [6]. It has been used as a test apparatus in computer aided analysis and design thus making it easier to understand and to experiment different control schemes ([3],[5]).

Mathematical models of mechanical systems are usually described by the Newtonion or Euler-Lagrange equations. These models have a structure which is very attractive for the design of control algorithms. This Euler-Lagrange approach has already been proved to be effective in the control design for mechanical and electro –mechanical system [7]. In this paper this method is utilized to investigate global properties of the designed controller.

2. MECHANICAL CONSTRUCTION

The system comprises of a horizontal plate that is connected to two wheels through a connecting rod. The wheels are independent of each other and are placed in the centre of the rail. Thus the platform can move on a horizontal surface and is able to rotate about the axis of wheels. There are two masses on top of the system that can slide along the horizontal rail, the masses being on both sides of the rail. The system is shown in fig(1)

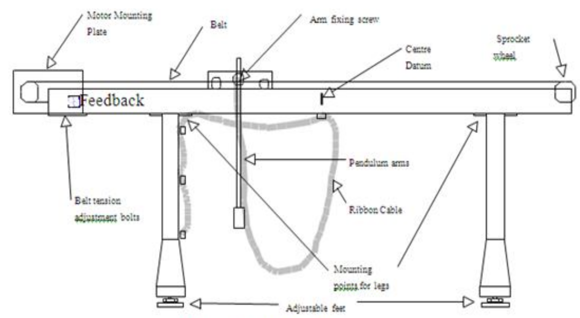

Fig 1: The pendulum system

3.MATHEMATICAL MODEL OF PHYSICAL SYSTEM :

The inverted pendulum is a classical problem in dynamics and control theory and is widely used as a benchmark for

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testing control algorithms' (PID controllers, SFB etc). Balancing the cart & pendulum is related to rocket or missile guidance where the centre of gravity is located behind the centre of drag causing aerodynamic instabity.

As a more realistic example of the design and simulation of controlled systems we now focus on the development and analysis of an inverted pendulum on a motor driven cart. A sketch of this system is shown in fig(2).

To study the system in detail we need its mathematical model. In this paper we find the mathematical model of the system using Euler-Lagrange's equations.

The resulting non-linear model is then linearized

The cart with an inverted pendulum, shown below is bumped with an impulse force F. The dynamic equation of motion for the system is linearized about the pendulum's angle, theta. The physical data [9] of the system are given in table no 1.

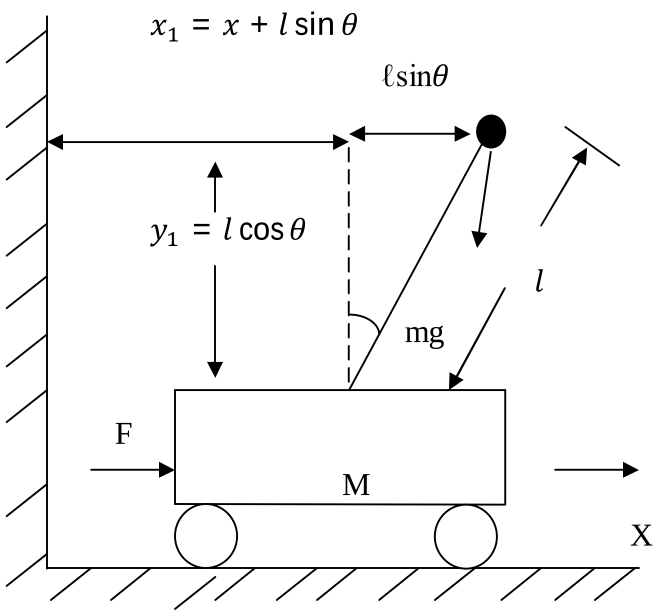

Fig 2 : The Inverted Pendulum System

Table 1. Parameters of the system from feedback instrument .U.K.

Parameter	Value	unit
Cart mass(M)	1.206	Kilo gram
Mass of the pendulum(N)	0.2693	Kilo gram
Length of pendulum(L)	0.1623	meter
Coefficient of frictional force(B)	0.005	Ns/m
Pendulum damping coefficient(D)	0.005	Mm/rad
Moment of inertia of pendulum(L)	0.099	Kg/m^2
Gravitation force(G)	9.8	m/s^2

F= control force;

X=displacement of the cart;

θ =pendulum angle from vertical;

Let V_1 be the resultant velocity

$$V_1^2 = \dot{x}_1^2 + \dot{y}_1^2$$

Therefore kinetic energy of the pendulum,

$$K_1 = \frac{1}{2} m V_1^2 + \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} m\,(\dot{x}^2 + 2l\dot{x}\dot{\theta}\cos\theta + ml^2\dot{\theta}^2] + \frac{1}{2} I\dot{\theta}^2$$

Kinetic energy of the of the cart,

$$K_2 = \frac{1}{2} M\dot{x}^2,$$

Potential energy of the pendulum,

$$P_1 = mgl\cos\theta$$

Potential energy of the cart P_2=0

The Lagrangian of the entire system is given as,

$$L = \frac{1}{2}(m\dot{x}^2 + 2ml\dot{x}\dot{\theta}\cos\theta + ml^2\dot{\theta}^2 + M\dot{x}^2) + \frac{1}{2} I\dot{\theta}^2] - mgl\cos\theta$$

The Euler-Lagrange's equation for the cart &resultant system is given as

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right) - \frac{\delta L}{\delta x} + b\dot{x} = F$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}}\right) - \frac{\delta L}{\delta \theta} + d\dot{\theta} = 0$$

Using these two above equations and putting the system parameters value we get,

$$(I + ml^2)\ddot{\theta} + ml\cos\theta\,\ddot{x} - mgl\sin\theta + d\dot{\theta} = 0$$

$$(M + m)\ddot{x} + ml\cos\theta\,\ddot{\theta} - ml\sin\theta\,\dot{\theta}^2 + b\dot{x} = F$$

The above equation shows the dynamics of the system.

4. LINEAR DIFFERENTIAL EQUATION MODELING

In order to derive the system transfer functions, we need to liberalize the differtional equation obtained so far. For small angle deviation around equilibrium point .When pendulum is in upright position $\sin\theta = \theta, \cos\theta = 1, \ \dot{\theta}^2 = 0$

Using above relation we can write as,

$r\ddot{\theta}+q\ddot{x}-k\theta+d\dot{\theta}=0$

$p\ddot{x} + q\ddot{\theta} + b\dot{x} = F$

Where , (M+m)= p, mgl=k, ml=q, $I+ml^2=r$

5 Transfer Function Modelling:

After taking Laplace transform of linear differential equation we get the following T.F. model,

$$\frac{\theta(s)}{F(s)} = \frac{-qs^2}{rs^2 - k + ds}$$

$$\frac{\theta(s)}{F(s)} = \frac{-0.2783\, S^2}{s\,(s + 2.026)\,(s - 1.978)\,(s + 0.03402)}$$

&

$$\frac{X(s)}{F(s)} = \frac{rs^2 - k + ds}{(pr-q^2)s^4 + (pd+br)s^3 + (bd-pk)s^2 - kbs}$$

$$\frac{X(s)}{F(s)} = \frac{0.68843\,(s + 2.014)\,(s - 1.967)}{s\,(s + 2.026)\,(s - 1.978)\,(s + 0.03402)}$$

6 State Space Modeling:

Let, $\theta = x_1$, $\dot{\theta} = x_2 = \dot{x}_1$, $\ddot{\theta} = \dot{x}_2 = \ddot{x}_1$ and, $x = x_3$, $\dot{x} = x_4 = \dot{x}_3$, $\ddot{x} = \dot{x}_4 = \ddot{x}_3$
From state space modeling, the system matrices are found in vector from given below.

$$A=\begin{bmatrix} 0 & 1 & 0 & 0.0000000 \\ 4.0088 & -0.047753952 & 0 & 0.0139155 \\ 0 & 0 & 0 & 1.0000000 \\ 0.116806166 & 0.0939155 & 0 & 0.0344200 \end{bmatrix}$$

and

$$B=\begin{bmatrix} 0 \\ -0.27831 \\ 0 \\ 0.68842 \end{bmatrix}$$

The output equations are

$$Y=\begin{bmatrix} 1\,0\,0\,0 \\ 0\,0\,1\,0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

7. PID Controller design:

The mathematical equation for PID control is

$$e_a(t) = K_p e(t) + K_d \frac{d_e(t)}{dt} + K_i \int_0^t e(\tau)\, d\tau$$

The block diagram of the whole system is shoen in fig (3)

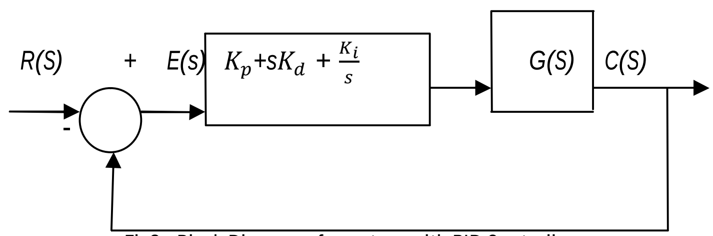

Fig3:- Block Diagram of a system with PID Controller

8. State Feedback Controller Design:-

CONDITIONS:

Our problem is to have a closed loop system having an overshoot of 10% and settling time of 1 sec. Since the overshoot

$M_P = e^{-\pi\xi\sqrt{1-\xi^2}}$ =0.1.

Therefore, ξ = 0.591328 and w_n=6.7644 rad/sec. The dominant poles are at=-4 ± j5.45531, the third and fourth pole are placed 5 & 10 times deeper into the s-plane than the dominant poles. Hence the desired characteristics equation:-

$s^4 + 68s^3 + 845.7604s^2 + 9625.6s + 36608.32 = 0$

Let gain, k= [k_1 k_2 k_3 k_4]

$$A - Bk = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ -k_1 & 0.2346-k_2 & 6.8963-k_3 & -0.0765-k_4 \end{pmatrix}$$

Closed loop characteristics equation:-

$S^4 + (0.0765 - k_4)s^3 + (-6.8963 + k_3)s^2 + (-0.2346 + k_2)s + k_1 = 0$

Comparing all the coefficient of above equation we found.

K = [36608.32 9625.8346 852.6567 -67.9235]

9. Simulation and Results:-.

9.1-Stabilisation of position of the cart with PID controller.

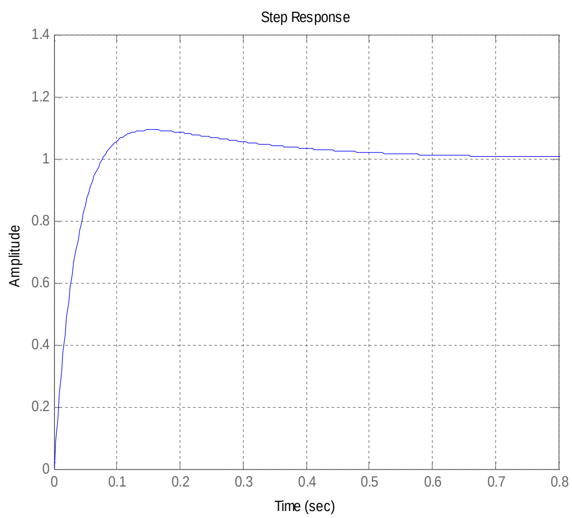

Fig4(a) : step response of the system when position as output for small time

For stabilization we found the PID controller parameters are,

K_d: - 45 K_P: - 200 K_I: - 24.2

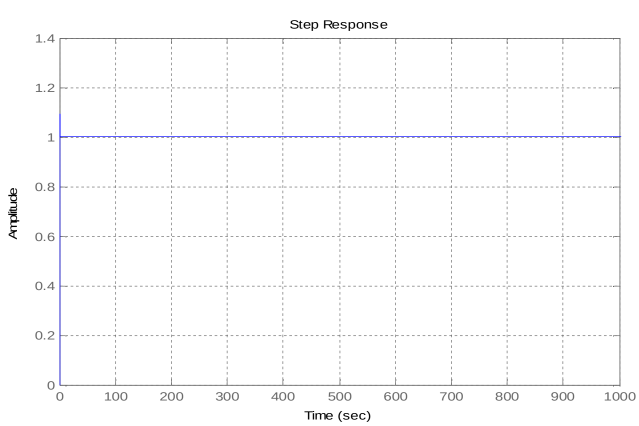

Fig4(b): step response of the system when position as output for large time

9.1.1-Stabilisation of angel of the cart with PID controller.

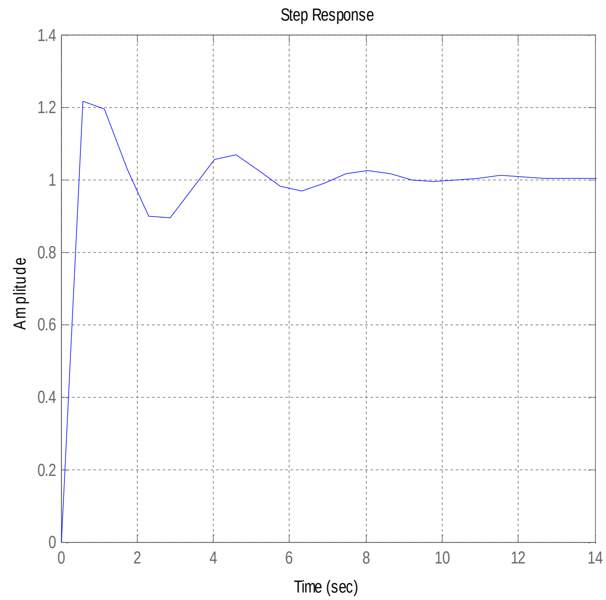

Fig4(c): step response of the system when angle as output for small time

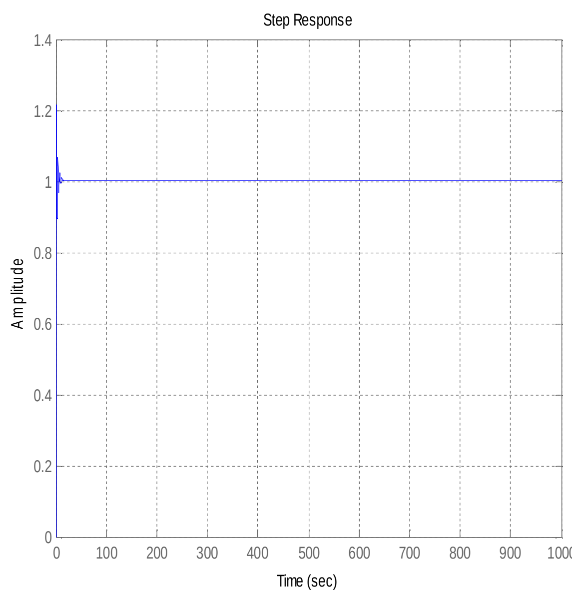

Fig4(d):- step response of the system when angle as output for small time and large time

For stabilization we found the PID controller parameters are,

K_D=-50 k_p= -58 K_i = -150

9.2-Stabilisation of Angle of the Pendulum by State Feedback Controller with Initial Condition:-

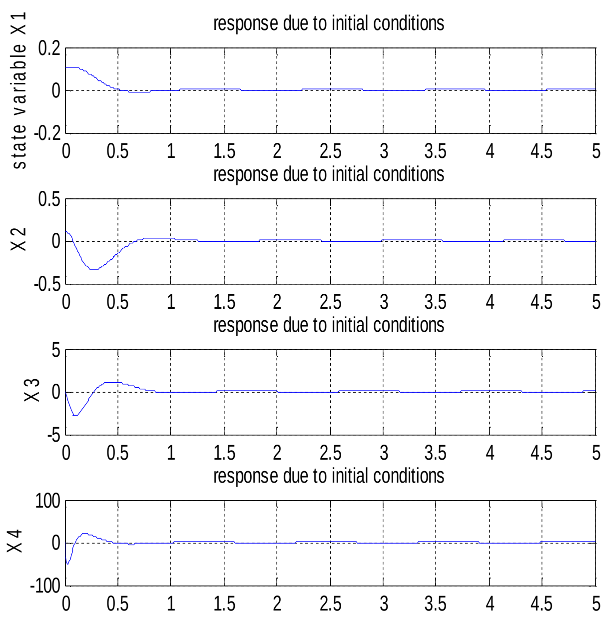

Fig5.7:- Response of state feedback controller considering initial condition in MATLAB

9.2.1-Step Response of the System by State Feedback Controller:-

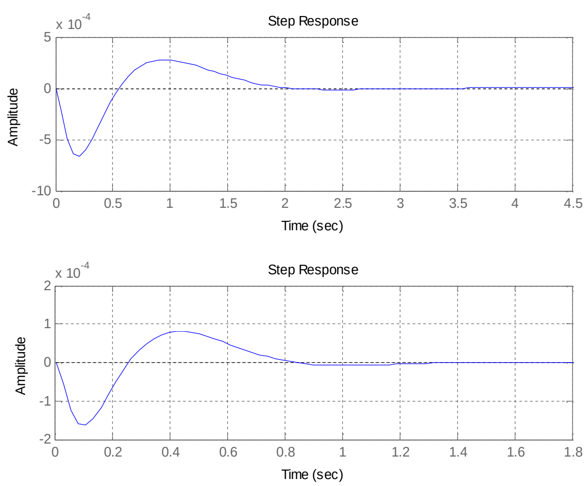

Fig5.8:- Response of Transfer Function represents Angle as output using state feedback controller considering Step input in MATLAB

10. Conclusion:-

Modelling of inverted pendulum shows that system is unstable with non-minimum phase zero. Results of applying two different controllers (PID & SFB) show that the system can be stabilized. while PID controller method is cumbersome because of selection of constants of controller, the controller parameters in case of state feedback are chosen analytically. Constant of the controllers can be tuned by some optimization technique for better result.

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